

Determining the current and voltage maxima on a *Lecher* line

Objects of the experiments

- Generating standing decimeter waves on a *Lecher* line with short-circuited end, with open end and with adjusted terminating resistor.
- Determining the wavelength λ from the distances between the current and voltage maxima.

Principles

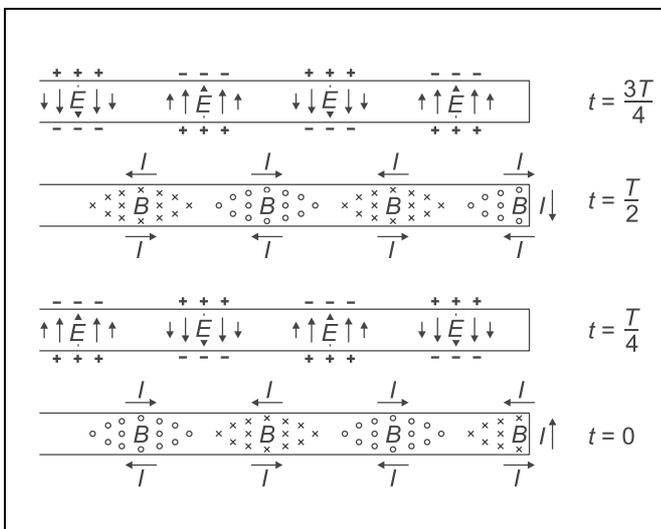
In 1890, *E. Lecher* proposed an arrangement of two parallel wires with circular cross-section to study the propagation of electromagnetic waves. When a high-frequency electromagnetic field is transmitted onto such a *Lecher* line, a voltage wave

$$U = U_0 \cdot \sin(\omega t - kx) \quad (I)$$

with $\omega = 2\pi\nu$, $k = \frac{2\pi}{\lambda}$

propagates in the direction x of the wires. The frequency ν and the wavelength λ of this wave agree with those of the transmitted field.

Standing electromagnetic waves on a *Lecher* line and the corresponding currents I and charges (+, -) on the wires.



Lecher line with short-circuited end:

If the wires of the *Lecher* line are short-circuited at the end, then the voltage U is zero. A reflected wave arises with a phase shift of 180° with respect to the incoming wave. For example, a voltage wave

$$U_1 = U_0 \cdot \sin(\omega t - kx)$$

coming in from the left together with a jumper at $x = 0$ gives rise to the reflected wave

$$U_2 = -U_0 \cdot \sin(\omega t + kx).$$

Both waves interfere to form the standing wave

$$U = U_1 + U_2 = -2 \cdot U_0 \cdot \sin kx \cdot \cos \omega t \quad (II).$$

The voltage between the wires is associated with a charge distribution along the wires. The displacement of these charges leads to a current I in the wires which propagates as a wave. There must be a permanent current at the jumper. The incoming current wave

$$I_1 = I_0 \cdot \sin(\omega t - kx)$$

is therefore reflected without a change in phase; that is, the reflected current wave has the form

$$I_2 = I_0 \cdot \sin(\omega t + kx).$$

I_1 and I_2 interfere to form the standing wave

$$I = I_1 + I_2 = 2 \cdot I_0 \cdot \cos kx \cdot \sin \omega t \quad (III).$$

Eqs. (II) and (III) show that the nodes of the voltage wave just correspond to the antinodes of the current wave and the antinodes of the voltage wave to the nodes of the current wave. The positions of the voltage nodes are

$$x = 0, \frac{\lambda}{2}, -\lambda, -\frac{3\lambda}{2}, \dots \quad (IV);$$

that is, their distances from the end of the wires are multiples of $\frac{\lambda}{2}$.

Apparatus

1 UHF transmitter	587 55
1 plug-in unit 230 V/12 V	562 791
1 Lecher system with accessories	587 56
1 steel tape measure, 2 m	311 77
3 saddle bases	300 11

Open ended Lecher line:

The situation changes, when the end of the *Lecher* line is left open. Now there is a permanent voltage at the end; that is, the incoming voltage wave is reflected without a change in phase, whereas the current at the open end is always zero, the incoming current wave being reflected with a phase shift of 180°. For the arising standing waves the equations

$$U = U_1 + U_2 = 2 \cdot U_0 \cdot \cos kx \cdot \sin \omega t \quad (V)$$

and

$$I = I_1 + I_2 = -2 \cdot I_0 \cdot \sin kx \cdot \cos \omega t \quad (VI),$$

follow. They can be obtained from Eqs. (II) and (III) by mutual exchange of U and I .

Lecher line terminated with the characteristic wave impedance:

Standing waves do not arise if the ends of the *Lecher* line are connected by an ohmic resistance which is equal to the characteristic wave impedance of the *Lecher* line. In this case, incoming current and voltage waves are not reflected at the end of the line.

Safety notes

Experiment setups using the UHF transmitter may not always conform to the limit values of class A (group 2 of the standard EN 55011). The device can interfere with other equipment in the experiment room of the educational facility. Also, radio interference can occur up to a distance of several hundred meters. It is the responsibility of the user to take all precautions to ensure that equipment installed outside of the experiment room can continue to function properly.

- See the information contained in the Instruction Sheet of your UHF transmitter.
- Do not operate the transmitter longer than is required to conduct the experiment; when the experiment is concluded, shut down the device immediately by switching off the plug-in supply unit.

Detection of standing waves:

In the experiment, the propagation of decimeter waves ($\nu = 433,92$ MHz) along a *Lecher* line is studied.

The standing voltage wave is detected by means of a probe consisting of a lamp that connects two conducting pins. These pins are slid along the *Lecher* line at a fixed distance. At the voltage antinodes, the voltage between the two conducting pins has its maximum value, and the lamp lights up brightly.

The standing current wave is detected by means of an induction loop connected to a lamp. At the current antinodes, the lamp lights up brightly because the magnetic field generated between the wires of the *Lecher* line oscillates at maximum amplitude.

Setup

The experimental setup is illustrated in Fig. 1.

- Clamp the UHF transmitter in the saddle base and choose operating mode CW.
- Plug the sections of the Lecher line together, slide on a holder with rod from one end and clamp it in a base.
- Plug the 4-mm plugs of the Lecher line into the antenna output of the UHF transmitter.
- Align the UHF transmitter and the Lecher line in height so that the Lecher line is horizontal.
- Slide a holder with rod on the coupling loop from one end, and clamp it in a saddle base.
- Make an induction loop by putting the lamp socket E10 together with the lamp (2) on the coupling loop (1).
- Align the induction loop in height so that the curved end is somewhat higher than the Lecher line.

Carrying out the experiment**a) Lecher line with short-circuited end:**

- Switch on the UHF transmitter by plugging in the plug-in unit.
- Plug the bridging plug into the open end.
- Slide the induction loop along the Lecher line, and find a position where the lamp lights up as brightly as possible.
- Optimize the brightness by shifting the induction loop perpendicularly to the Lecher line, and, if necessary, by carefully reducing its distance from the Lecher line without touching the Lecher line.
- Slide the induction loop along the Lecher line, first “from the left to the right”, then “from the right to the left”. Make marks on the Lecher line at all points (current nodes) where the lamp just goes out.

The current nodes are each in the middle of a “left” and a “right” mark.

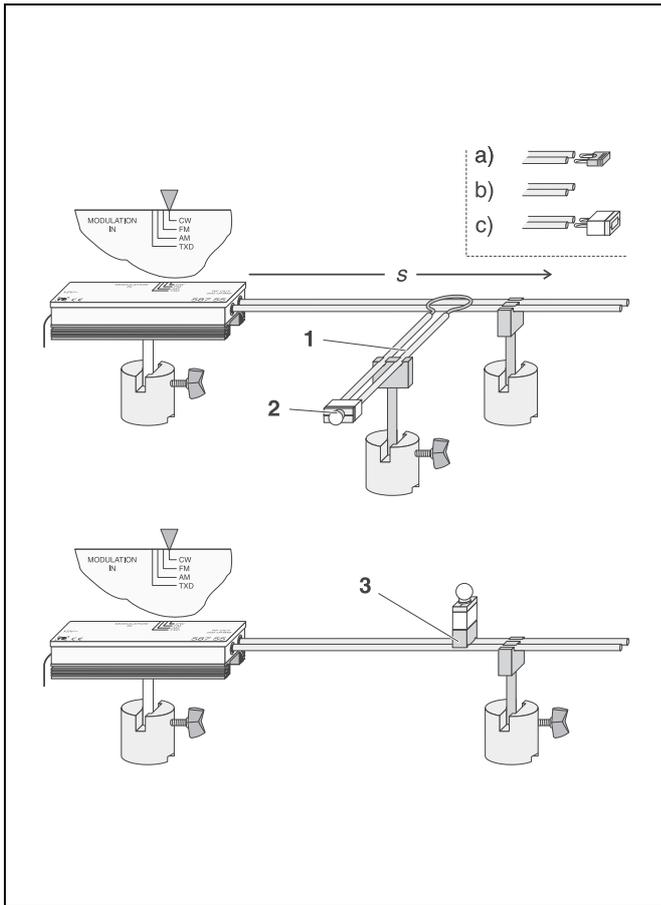


Fig. 1 Experimental setup for the detection of the current maxima and minima (above) and for the detection of the voltage maxima and minima (below)

- Assemble the plastic adapter (3) and the lamp socket E10 with lamp (2) and place it on the Lecher line as a probe.
- Slide the probe along the Lecher line, first "from the left to the right", then "from the right to the left". Make marks on the Lecher line at all points (voltage nodes) where the lamp just goes out.
The voltage nodes are each in the middle of a "left" and a "right" mark.
- Measure the positions s of the current and voltage nodes on the Lecher line with the steel tape measure starting from the output of the UHF transmitter and record them.
- Use the probe to detect the voltage antinodes at the positions of the current nodes.
- Use the induction loop to detect the current antinodes at the positions of the voltage nodes.

b) Open ended Lecher line:

- Use the probe to find the voltage nodes on the Lecher line, and mark them.
- Use the induction loop to find the current nodes on the Lecher line, and mark them.
- Measure the positions s of the current and voltage nodes on the Lecher line with the steel tape measure starting from the output of the UHF transmitter and record them.
- Use the probe to detect the voltage antinodes at the positions of the current nodes.
- Use the induction loop to detect the current antinodes at the positions of the voltage nodes.

c) Lecher line terminated with the characteristic wave impedance:

Caution: The sustained load capacity of the 200 Ω terminator is only 2 W:
Do not leave the 200 Ω terminator attached for more than 5 minutes.

- Place the 200 Ω onto the open end.
- Look for brightness maxima and minima of the lamp with the induction loop and the probe.

Measuring example

Length s_0 of the Lecher line: 88 cm

a) Lecher line with short-circuited end:

Table 1: The positions s of the current and voltage nodes on the short-circuited Lecher line

N	$\frac{s}{\text{cm}}$	kind
1	5.0	current node
2	22.25	voltage node
3	38.5	current node
4	56.0	voltage node
5	72.75	current node
6	88.0	voltage node

b) Open ended Lecher line:

Table 2: The positions s of the current and voltage nodes on the open ended Lecher line

N	$\frac{s}{\text{cm}}$	kind
1	4.25	voltage node
2	21.25	current node
3	38.25	voltage node
4	55.25	current node
5	71.5	voltage node
6	88.0	current node

c) Lecher line terminated with the characteristic wave impedance:

No pronounced current or voltage nodes (or antinodes respectively) have been found.

Evaluation

Fig. 2 summarizes the measuring values from Tables 1 and 2 in a graph.

The straight line drawn into the graph agrees with the measuring values within the measuring accuracy; that is, the distances between the current and the voltage nodes are constant. The

slope of the straight line is 16,875 cm. From this the wavelength of the decimeter can be calculated:

$$\lambda = 4 \cdot a = 67.5 \text{ cm}$$

The phase velocity therefore is

$$v = \lambda \cdot \nu = 2.93 \cdot 10^8 \text{ m s}^{-1}.$$

Results

Decimeter waves propagate along a Lecher line. Standing waves can arise depending on the termination of the Lecher line.

If the end of the line is short-circuited, then there is a voltage node at the end. At an open end of a Lecher line there is a current node. If the end of the line is terminated with an ohmic resistor which is equal to the characteristic wave impedance Z , no standing waves arise because the supplied energy is consumed by the ohmic resistor.

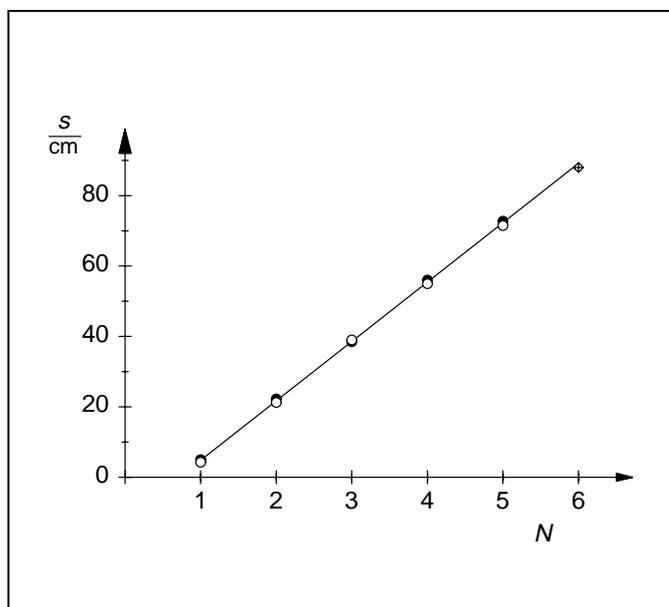
Fig. 2 The positions s of the voltage and current nodes plotted against the order N

closed circles: current nodes when the end is short-circuited

closed boxes: voltage nodes when the end is short-circuited

open circles: current nodes when the end is open

open boxes: voltage nodes when the end is open



Supplementary information

The value found for the phase velocity is a bit smaller than the vacuum velocity of light; that is, the wavelength is a bit contracted with respect to the wave length of the UHF transmitter. The reason for this contraction is the finite diameter of the Lecher line and the fact that, at high frequencies, currents flow on the surface and are not homogeneously distributed through the cross-section of the wires (skin effect).

The straight line drawn in Fig. 2 shows, by way of calculation, that the last (sixth) node is about 1.2 cm behind the end of the line ($s_0 = 88$ cm). This effect too is due to the finite diameter of the wires.

Moreover, a thorough consideration shows that the positions of the nodes for the short-circuited line are shifted by about 0.7 cm with respect to the nodes of the open ended line.