Four-Probe Resistance Primer

1. Introduction

If you wanted to determine the precise value of a resistor R, you would probably hook it's two leads up to an Ohmmeter and read off the value. Remember, however, that the resistance of the DMM leads will be in series with the resistance you are trying to measure, so will of course add to your answer. This could be a problem, say, if $R \leq 1\ \Omega$. Or what if the resistor you are trying to measure is very delicate? Suppose, for instance, that the resistor will "burn up" if you pass more than 10 μA of current through it. Do you dare connect it up to a DMM? How much current does the DMM use to measure resistance?

For these and other reasons, one often determines the resistance of a sample by passing through it a known current I, measuring the resulting voltage drop ΔV, and performing the division to get $R = \Delta V/I$. This might be a direct current, or it might be an alternating current. In the former case ΔV is measured with a sensitive voltmeter, in the latter it might be measured with an oscilloscope, or better yet, with a lock-in amplifier.

2. Constant-Current Source Using a Ballast Resistor

To calculate the sample resistance you must, of course, know the value of the current. If the current is quite small (say less than 1 μA, this can be a difficult task. Often you don't need to know the current with a high degree of accuracy, you only need to know that it doesn't change while you perform your series of measurements. For instance, if you are measuring the resistance of a Y-Ba-Cu-O compound as a function of temperature through its superconducting transition, you would be quite happy to measure ΔV(T) just knowing that I didn't change during the measurement.

The easiest way to establish a constant current through the sample is shown below. A known voltage E is connected to a series circuit consisting of the sample resistor R and a large, known resistor, R_B. The current is obviously just $E/(R_B + R)$. In general, if R changes (say, its temperature changes) so will the current. If we choose $R_B >> R$, however, then $I \approx E/R_B$, independent of changes in R. R_B is called the "ballast" resistor. If you are sure that $R_B >> R$, then I is known as long as E and R_B are known. For instance, you might establish a 10 μA through a sample having a resistance $R \approx 10\ \Omega$ by choosing E = 1 V and RB = 100 kΩ. Suppose the sample goes superconducting at some temperature (i.e., R = 0). By what amount does the current change? It is often convenient to use a decade resistor for R, both so that it can be easily changed, and also because the decade box is usually constructed from very stable resistors that do not change much with changes in room temperature..

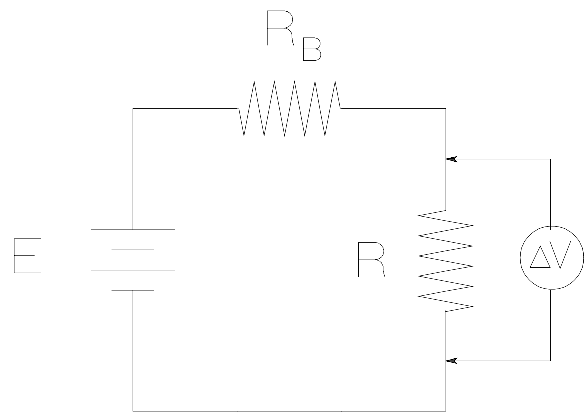

Figure 1: Constant-current circuit using a ballast resistor.

3. Contact Resistance

The constant-current circuit above allows us to determine the sample resistance with a very small current (assuming we are able to measure a small ΔV) eliminating the possibility of damage to the sample from the DMM. We now turn to the other issue, the problem of lead resistance. As mentioned above, the sample resistance might be so low that the resistance of the leads running to the sample might be significant by comparision. A related problem is that of contact resistance. Somehow we must connect leads between our sample and the external circuit, and this involves making "contact" to the sample. Contacts are notorious sources of resistance (and noise).1

The situation is illustrated below. Let the two contacts to the sample be represented by equivalent resistances R_1 and R_2. The measured voltage drop $V = I(R_1 + R + R_2)$. How do we know what fraction of the voltage drop V is due to R and how much is due to the contacts?2 Fact is we have no way of knowing, because we measure their *series combination*. This is especially a problem if R is much smaller than R_1 and R_2 (e.g., if R goes superconducting).

1 Moreover, as the contact involves an interface between two dissimilar materials, its I-V characteristics are frequently nonlinear, i.e., it may not be Ohmmic.

2 We can lump the lead resistance and contact interface resistance together.

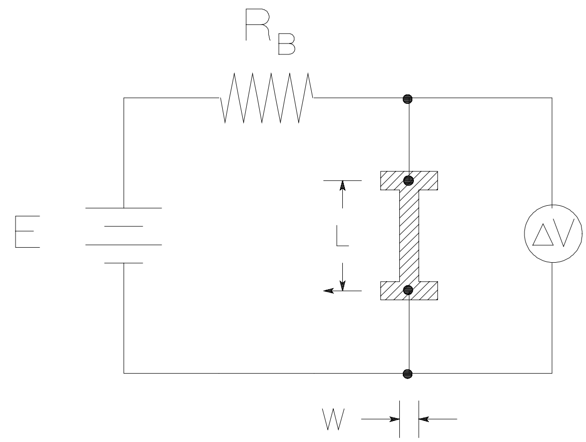

Figure 2: Constant-current circuit with a *two-probe* sample.

The standard way to separate out the sample resistance from the "contact" resistance is to use four, rather than two, sample contacts. This is illustrated in the figure below. By separating the current contacts from the voltage contacts we are able to distinguish the sample resistance from that of the contacts and connecting wires. This can be

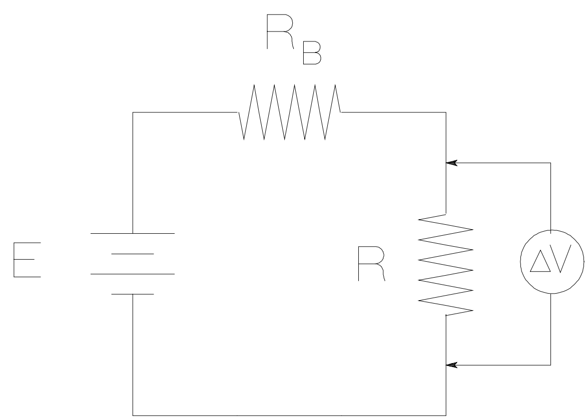

Figure 3: Circuit showing a *four-probe* resistance measurement.

seen by looking at the equivalent circuit, shown below. The current and voltage contacts are modeled as resistors R_1 - R_4. Electrically, current contact resistances R_1 and R_2 are effectively part of the ballast resistor R_B. If the voltmeter has an infinite input impedance, no current will flow through the voltage contacts R_3 and R_4, and the measured voltage drop V is across the portion of the sample that is between the two voltage contacts. Even if R is much smaller than R_1..R_4, the measured voltage drop is still V = IR.

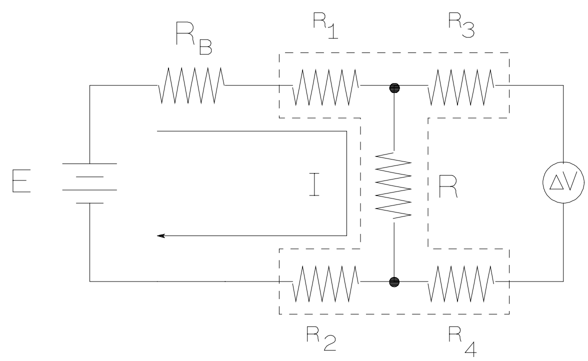

Figure 4: Equivalent circuit for the four-probe circuit of Figure 3.

4. Using a Lock-in to Measure Resistance

If you are trying to measure voltages on the order of microvolts, you should consider using a lock-in amplifier. Lock-in amplifiers are ideal for making low-frequency resistance measurements. The basic idea is to replace the battery E in Figure 3 with an oscillator $E_o\cos(2\pi f_o t)$, and to replace the voltmeter with a phase-sensitive-detector. Most lock-in amplifiers combine both of these. The oscillator frequency f_o is set to some low value, say 44 Hz. Figure 5 below shows a typical set-up. The lock-in is set to use its own internal oscillator as the reference for the PSD. The lock-in is calibrated to read ΔV in rms-voltage so that the sample resistance $R = \Delta V/I = (\Delta V/E_{rms})R_B$, where E_{rms} is the rms-voltage of the lock-in's oscillator.

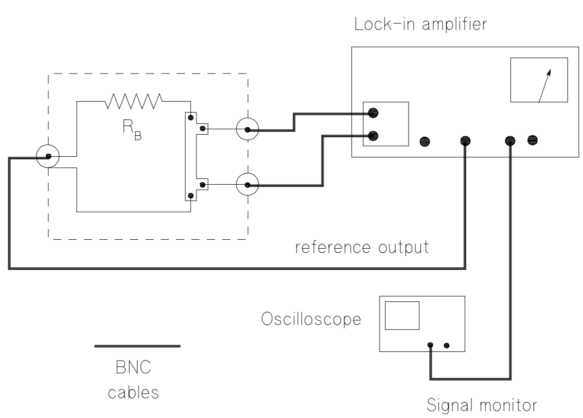

Figure 5: Four-probe resistance circuit with a lock-in amplifier.

In the circuit above there is an oscilloscope connected to the signal monitor output of the lock-in. It is very important to "look" at what it is that you are measuring -- never trust the reading without first viewing the signal. The dashed line around the sample is a metal shield. It is important (when possible) to surround all wires with an electrostatic shield to reduce 60 Hz pickup. Also, the BNC cable connections are "blown up" on the sample box to show the internal wiring of the box. To be sure, all BNC conectors are connected to the BNC cables as expected.

5. Four Probe Method for Determining Resistivity

We are accustomed to writing the resistance of a conductor as R = ρ(L/A), where L is the conductor length and A is its cross-sectional area. Built into this expression, however, is the assumption that the current density $\vec{j}$ is uniform throughout the conductor. Suppose the conductor was not of uniform cross-sectional area, but instead, had some narrow regions and some wider regions. You can easily convince yourself that this expression no longer holds. In general, the measured resistance is some weighted average of the resistivity over the volume of the conductor. The "weighting" is determined by the square of the current density, $\vec{j} \cdot \vec{j}$.

Consider what happens then, when current enters a conductor through. a "point" contact like those in Figure 2. The current density in the sample immediately under the contact is very large. "Downstream" the current quickly spreads and becomes fairly uniform. At the exit contact, the current again must "crowd" into the point contact. The "effective" sample resistance (even if it did not include lead and contact interface resistances) is not simply ρ(L/A), due to the nonuniform current density. Even if you were willing to integrate the weighting function, it is critically sensitive to the exact contact area, which is hard to determine.

The problem is avoided with a four-probe measurement like that of Figure 3. The situation at the current contacts has not improved. The improvement comes in that you measure the voltage drop "downstream" where the current density has become uniform. Now the resistance may be used to calculate the sample resistivity using the separation distance of the voltage probes for L.

There is one other important kind of four-probe resistivity measurement that you will find useful. This involves setting four, equally-spaced point contacts down on the surface of a "large" conductor, as shown in the Figure below. Let a be the probe spacing and h be the sample thickness. We assume that the sample is infinite (i.e., its horizontal dimensions are much larger than the probe spacing). A current I is passed through the sample via the outer two probes, and the voltage drop is measured between the inner two probes.

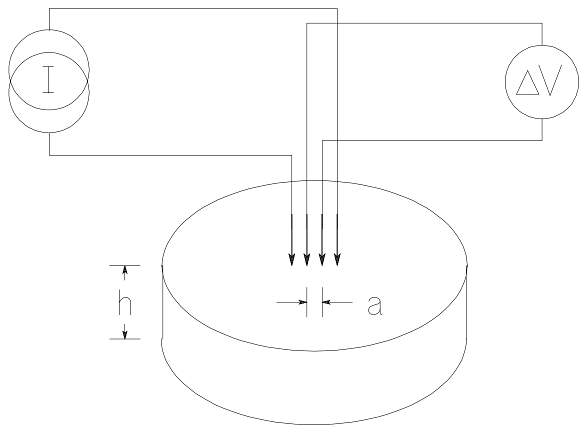

Figure 6: Four-probe method for measuring sheet resistance.

Consider two cases: 1) the sample is infinitely thick (i.e., h>>a), and 2) the sample is infinitely thin (i.e., h<<a). First, try to visualize the current-density lines for this situation. Lines of $\vec{j}$ look much like the electric field lines for a dipole in 3-dimensions for case 1, and 2-dimensions for case 2. Current enters the sample at the current contacts and quickly spreads. Undeneath the voltage contacts, the lines of $\vec{j}$ are determined, not by the nature of the contacts, but by the dimensionality of the conductor. For these two cases, the appropriate integrals have been performed to give the sample resistivity in terms of I and ΔV. The results are:

$$h << a: \quad \rho = \frac{\pi}{\ln(2)} h \left(\frac{\Delta V}{I} \right)$$

$$h >> a: \quad \rho = 2\pi a \left(\frac{\Delta V}{I} \right)$$

For the two-dimensional case, the quantity ρ/h (which has units of Ohms) is called the two-dimensional resistivity, sheet resistance, or resistance-per-square. In many thin film applications, one does not know the film thickness or resistivity, only the sheet resistance.

6. References

1. L. Maissel and R. Glang, *Handbook of Thin Film Technology* (McGraw-Hill, 1983), pp.13-5 to 13-7.
2. J. H. Scofield, *Review of Scientific Instruments* **58**, 985-993 (1987), "AC method for measuring low-frequency resistance fluctuation spectra."